B. Sc. Part I
Paper I

Dr. Shiva Kant Mishra
Dept. of Physics
H. D. Jain College Ara

R Theory of Relativity

[Since according to Lorentz Contraction $L = L_0\sqrt{\left(1-\frac{v^2}{c^2}\right)}$]

But there is no contraction in the direction of y-axis since motion of system S' is only along x-axis. So the length along y-axis in system S is 3 metre. Consequently the dimensions may be represented by $(4\hat{i} + 3\hat{j})$ metre.

A rocket of proper length 600 metre is moving directly away from earth. A light pulse sent from the earth is reflected from mirrors at the rear (back part) and front of the rocket. If the first of these reflected pulses is received back 100 sec after emission and the second one 8 μ-sec later, calculate:

a) the distance of the rocket from earth.

b) the velocity of the rocket and c) its apparent length.

Sol:- (a)

The pulse sent from the earth reaches the rear of the rocket 50 sec after its emission, hence at this instant the distance of the rear of the rocket from earth

$$= 50 \times 3 \times 10^8 \text{ metres} = 1.5 \times 10^{10} \text{ metre}$$

(b) According to Lorentz transformation in differential form

$$\delta t = \frac{\delta t' + \frac{v\,\delta x'}{c^2}}{\sqrt{(1-\beta^2)}}$$

If the events of reception of light use at the rear and front of the rocket be characterised by (x_1, t_1) and (x_2, t_2) in earth frame S and (x_1', t_1') and (x_2', t_2') in the rocket frame S' moving relative to S along +ve common x-axis, Then given

$$\delta x' = x_2' - x_1' = 600 \text{ metre}$$

$$\delta t' = t_2' - t_1' = \frac{x_2' - x_1'}{c} = \frac{\delta x'}{c}$$

and $\delta t = t_2 - t_1 = 8 \mu sec = 8 \times 10^{-6}$ sec

$$\therefore \quad \delta t = \frac{(1+\frac{v}{c})\delta x'}{c\sqrt{(1-v^2/c^2)}} \quad \underline{\underline{or}} \quad \sqrt{\left(\frac{1+v/c}{1-v/c}\right)} = \frac{c\delta t}{\delta x'}$$

$$= \frac{3\times10^8 \times 8 \times 10^{-6}}{600} = 4$$

$$\therefore \quad v = 0.88c.$$

(c) The apparent length of rocket is

$$l = 600\sqrt{\left(1-\frac{v^2}{c^2}\right)}$$

$$= 600 \times \sqrt{[1-(0.88)^2]} = \underline{\underline{280 \text{ metre}}}$$

Problem Calculate the length of a rod moving with a velocity of 0.8c in a direction inclined at 60° to its own length. Given proper length of the rod = 1 metre.

Sol:- If l_0 is the proper length of the rod and 0.8c is its velocity along x-axis, we have

$$l_0 = l_0 \cos 60^\circ i + l_0 \sin 60^\circ j$$

where i and j are unit vectors along X and Y axes respectively.

As the contraction takes place only in the direction of motion the contraction will take place along X-axis, while the component of length along Y-axis will remain the same.

If l_x' is the component of the length of the rod, when in motion along x-axis we have

$$l_x' = l_x\sqrt{(1-\beta^2)} = l_0 \cos 60^\circ \sqrt{(1-v^2/c^2)}$$

$$= l_0 \cos 60^\circ \sqrt{\left[1-\left(\frac{0.80c}{c}\right)^2\right]} = 0.3 l_0$$

The Component of Moving rod along y-axis

$$l'_y = l'_0 \sin 60^\circ = \sqrt{3}/2\, l_0$$

$\therefore$ The length of the Moving rod

$$= \sqrt{(l_x'^2 + l_y'^2)}$$

$$= \sqrt{\left[(0.3 l_0)^2 + \left(\frac{\sqrt{3}}{2} l_0\right)^2\right]} = 0.9\, l_0$$

Given $l_0 = 1$ metre.

$\therefore$ The length of the rod in Motion $= 0.91 \times 1$

$= 0.91$ metre.

Problem :-

Calculate the length and orientation of a rod of length 5 Metre in a frame of reference which is Moving with velocity $0.6c$ in a direction making an angle 30° with the rod.

Sol :-

Given Proper light $l = 5$ metre. According to length Contraction, the Contraction takes trace only along to direction of Motion; while Perpendicular to direction of Motion, no Contraction taken Place.

Component of length of rod along to direction of Motion

$$l_x = l\cos 30^\circ = 5 \times \sqrt{3}/2 = 4.33\,m$$

Component of length of rod Perpendicular to direction of Motion

$$l_y = l \sin 30^\circ = 5 \times \frac{1}{2} = 2.5\,m$$

... from Moving frame